

**Spring 2020, Math 621: Preliminary Problem Set 5**  
**Due: Thursday, February 27th, 2020**  
**Hilbert's Theorem and Quasipolynomials**

**Preliminary problems.** These problems should be completed before discussion on Thursday.

(P1) Consider the ideal  $I = \langle x^2 - xy, xy - y^2 \rangle \subset R = \mathbb{k}[x, y]$ .

(a) Draw the staircase diagram of  $I$ .

(b) Find  $\text{Hilb}(R/I; z)$ , and use it to find  $\text{Hilb}(I; z)$  (both under the standard grading).

(c) What does the Hilbert function of  $I$  appear to be counting in the staircase diagram?