

Spring 2021, Math 522: Problem Set 0
Due: Thursday, January 28th, 2021
Proof Writing Review

Homework problems. You must submit *all* homework problems in order to receive full credit.

(H1) Suppose $x, y \in \mathbb{R}$. Prove $x^3 + x^2y = y^2 + xy$ if and only if $y = x^2$ or $y = -x$.

(H2) Suppose $x \in \mathbb{R}$ with $x > 0$.

(a) Use induction on n to prove that

$$(1 + x)^n \geq 1 + nx$$

for all $n \in \mathbb{Z}$ with $n \geq 1$.

(b) Is part (a) still true if the “ $x > 0$ ” assumption is omitted? If so, where in your proof did you use this assumption?

(H3) Prove that $\log_2(3)$ is irrational.

(H4) Fix a set A and a relation R on A . Consider the following **false** statement.

“If R is symmetric and transitive, then R is reflexive.”

Articulate the error in the following “proof” of the above statement.

Proof. Since R is symmetric, $(a, b) \in R$ implies $(b, a) \in R$ for any $a, b \in A$. Since R is transitive, $(a, b) \in R$ and $(b, a) \in R$ together imply $(a, a) \in R$. As such, $(a, a) \in R$ for all $a \in A$, so we conclude R is reflexive. \square

(H5) Determine whether each of the following statements is true or false. Prove your claims.

(a) The function $f : \mathbb{R} \rightarrow \mathbb{R}$ given by

$$f(x) = \frac{x}{x^2 + 1}$$

is injective.

(b) The function $f : \mathbb{Z} \rightarrow \mathbb{R}$ given by

$$f(x) = \frac{x}{x^2 + 1}$$

is injective.