Spring 2021, Math 621: Preliminary Problem Set 9
Due: Thursday, March 25th, 2021
Venturing into Homological Algebra

Preliminary problems. These should be submitted to Gradescope before Thursday discussion.

(P1) Consider the (chain) complex $C_\bullet$ of the form

$$\cdots \longrightarrow \mathbb{Q}^2 \xrightarrow{\partial_2} \mathbb{Q}^2 \xrightarrow{\partial_1} \mathbb{Q}^2 \xrightarrow{\partial_2} \mathbb{Q}^2 \xrightarrow{\partial_1} \cdots$$

where

$$\partial_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \qquad \text{and} \qquad \partial_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

Demonstrate this is a complex, and determine whether it is exact everywhere (that is, determine whether $\ker(\partial_1) = \mathrm{Im}(\partial_2)$ and $\ker(\partial_2) = \mathrm{Im}(\partial_1)$.