

**Spring 2022, Math 579: Preliminary Problem Set 7**  
**Due: Thursday, March 10th, 2022**  
**Introduction to Generating Functions**

**Preliminary problems.** These problems should be completed before discussion on Thursday.

(P1) Let

$$A(z) = \sum_{n=1}^{\infty} z^n = z + z^2 + z^3 + z^4 + \dots .$$

Note the first term!

- (a) Fill in the coefficients below for the product  $A(z)A(z)$ . Show enough work to easily recall where each term comes from.

$$\begin{aligned} (A(z))^2 &= (z + z^2 + z^3 + \dots)(z + z^2 + z^3 + \dots) \\ &= \underline{\hspace{2cm}} + \underline{\hspace{2cm}}z + \underline{\hspace{2cm}}z^2 + \underline{\hspace{2cm}}z^3 + \underline{\hspace{2cm}}z^4 + \dots \end{aligned}$$

- (b) Fill in the coefficients below for the composition  $A(A(z))$ . Show enough work to easily recall where each term comes from.

$$\begin{aligned} A(A(z)) &= A(z) + (A(z))^2 + (A(z))^3 + \dots \\ &= \underline{\hspace{2cm}} + \underline{\hspace{2cm}}z + \underline{\hspace{2cm}}z^2 + \underline{\hspace{2cm}}z^3 + \underline{\hspace{2cm}}z^4 + \dots \end{aligned}$$