Fall 2022, Math 621: Preliminary Problem Set 4
Due: Thursday, February 17th, 2022
Graded Modules

Preliminary problems. These problems should be completed before discussion on Thursday.

(P1) Suppose R is a ring graded by a semigroup T, and suppose M is an R-module.

(a) Write the definition of a grading of M by T (this was in the notes from Tuesday).

(b) Define the Hilbert function of M (this is new, but your best guess is probably right).

(P2) For each of the following rings R and semigroups T, specify a grading of R by T in which each graded piece is dimension at most 1 (we call this a *fine* grading (not to be confused with **the** fine grading, which is a particular fine grading)). Write each graded piece of R as a span of monomials, and write the Hilbert function of R.

(a) $R = \Bbbk[x]/\langle x^3 \rangle$, $T = \mathbb{Z}$

(b) $R = \Bbbk[x]/\langle x^3 - 1 \rangle$, $T = \mathbb{Z}_3$