

**Fall 2022, Math 621: Preliminary Problem Set 9**  
**Due: Thursday, March 24th, 2022**  
**Hilbert's Theorem and Quasipolynomials**

**Preliminary problems.** These problems should be completed before discussion on Thursday.

(P1) Find the dimension of each of the following modules  $M$  over  $R = \mathbb{k}[x, y]$ .

(a)  $M = R/\langle x - y \rangle$

(b)  $M = R^2$

(c)  $M = (R/\langle x \rangle) \oplus (R/\langle y \rangle)$

(P2) If  $M_1$  and  $M_2$  are  $R$ -modules, how is  $\dim(M_1 \oplus M_2)$  related to  $\dim(M_1)$  and  $\dim(M_2)$ ?