## Fall 2022, Math 621: Preliminary Problem Set 10 Due: Thursday, April 7th, 2022 <br> Gröbner Bases

Preliminary problems. These problems should be completed before discussion on Thursday.
(P1) Recall that in class, we divided

$$
f=x^{4} y^{2}+x y^{2}-2 x \quad \text { by } \quad g_{1}=x y^{3}+x^{2} y-2 \quad \text { and } \quad g_{2}=x^{2}-y
$$

using glex order. Perform polynomial long division again under glex order, but with $g_{2}$ listed before $g_{1}$. You should obtain a different remainder than we obtained in class.
(P2) Use Buchberger's algorithm to obtain a Gröbner basis for

$$
I=\left\langle x^{2}-y^{2}, x^{4} y-x y^{3}\right\rangle \subseteq \mathbb{k}[x, y]
$$

under the glex term order. Your answer should consist of 3 polynomials.

