Math 245 - Spring 2026
Instructor: Christopher O'Neill
Friday, February 20th, 2026
Midterm Exam 1

Name: _____ **Red ID:** _____

Please write legibly, with plenty of white space. Please **print** your name and Red ID in the designated spaces above. Be sure to read problem directions carefully. Please fit your answers into the designated areas; material outside the designated areas (such as on this cover page) will not be graded. All problems are worth 10 points, for a maximum score of 100. The use of notes, books, calculators, or other materials on this exam is strictly prohibited, except you may bring one 3"x5" card (both sides) with your handwritten notes. This exam will begin at 10:00 and will end at 10:50; pace yourself accordingly.

Logistical instructions:

(1) Stow all bags/backpacks/purses at the front of the room. All contraband, except phones, must be stowed in your bag. All smartwatches and phones must be powered down and either in your pocket or stowed in your bag.
(2) Please remain quiet to ensure a good test environment for others.
(3) Please keep your exam on your desk; do not lift it up for a better look.
(4) If you have a question or need to use the restroom, please come to the front. Bring your exam. I cannot come to you unless you are sitting by an aisle.
(5) If you are done and want to submit your exam and leave, please wait until one of the three designated exit times below. Please do **NOT** leave at any other time. If you are sure you are done, just sit and wait until the next exit time, with this cover sheet visible.

Good luck!

Designated exit times: 10:20 10:40 10:50

(1) Fill in the blanks in each of the following. Unless otherwise stated, p, q, and r are arbitrary propositions. No justification is required for this problem.

(a) If a well-formed expression has free variable, then that expression is a _____.

If each variable is bound by a _____, then that expression is a _____.

(b) For $x \in \mathbb{Z}$ arbitrary, "$x = \sqrt{2} \ \rightarrow \ x = 5$" is best proven using a _____ proof.

(c) For $x \in \mathbb{Z}$ arbitrary, "$2 \mid x \ \rightarrow \ x \cdot 0 = 0$" is best proven using a _____ proof.

(d) If p is an arbitrary proposition, then truth tables can be used to prove the compound proposition

$p \vee \neg p$ is a _____, while the compound proposition $p \wedge \neg p$ is a _____.

(e) The negation of "$\forall x \in \mathbb{Z}, \exists y \in \mathbb{R}, xy = 0$" is equivalent to "_____, $xy \neq 0$".

(2) Fix arbitrary propositions p, q, r, and s, and consider the proposition

$$(p \wedge \neg q) \rightarrow (r \vee \neg s).$$

Write "Y" next to each of the following that are equivalent to the above proposition, and write "N" next to those that are not equivalent to the above proposition. No justification is required.

_____ $(\neg p \wedge q) \rightarrow (\neg r \vee s)$ _____ $(r \vee \neg s) \rightarrow (p \wedge \neg q)$

_____ $(\neg r \vee s) \rightarrow (\neg p \wedge q)$ _____ $(\neg r \wedge s) \rightarrow (\neg p \vee q)$

(3) Label each of the following as either "true", "false", or "neither". No justification is required.

_____ "There exists $x \in \mathbb{Z}$ such that $x - y \geq 5$"

_____ "For all $x \in \mathbb{Z}$, there exists $y \in \mathbb{Z}$ such that $x - y \geq 5$"

_____ "There exists $x \in \mathbb{Z}$ such that for all $y \in \mathbb{Z}$, $x - y \geq 5$"

_____ "For all $x \in \mathbb{Z}$ and all $y \in \mathbb{Z}$, $x - y \geq 5$"

(4) Fill in the blanks in the following proof that for all $x \in \mathbb{R}$, if x^2 is irrational, then $3x$ is irrational.

Proof: _____. We use a contrapositive proof. Suppose _____.

Then there exist $a, b \in \mathbb{Z}$ with $b \neq 0$ such that _____ = _____. Substitution yields

_____ = _____,

so since _____ $\neq 0$, we conclude _____. $\square$

(5) Consider the following improperly written definition.

> "A *root* is when a you plug a number into a polynomial and get 0."

Using the above as a guide, write a proper mathematical definition for the term *root*.

(6) Fix arbitrary propositions p, q, and r. Determine whether each of the following equivalences is valid. Prove your claims **without using truth tables**.

(a) $(p \vee q) \wedge r \equiv p \vee (q \wedge r)$

(b) $(p \to q) \vee r \equiv \neg p \vee (q \vee r)$

(7) Fix arbitrary propositions p, q, r. **Without truth tables**, prove the semantic theorem

$$\neg q \to \neg p, p \vee r \vdash q \vee r.$$

(8) Prove that $\forall x \in \mathbb{N}, \exists y \in \mathbb{Z}, \forall z \in \mathbb{R}, x > yz$.

(9) Prove or disprove: for all $a, b \in \mathbb{Z}$, if a is odd and b is even, then $a^2 + ab$ is even.