(5) Fix two subsets A and B of a universe set U.

(a) Prove $(A^c \setminus B^c) \subseteq (A \setminus B)^c$.

Fix $x \in A^c \setminus B^c$. This means $x \in A^c$ and $x \notin B^c$. As such, $x \in U$ and $x \notin A$. Since $A \setminus B \subseteq A$, and $x \notin A$, $x \notin A \setminus B$. This means $x \in U \setminus (A \setminus B) = (A \setminus B)^c$ as desired.

(b) Disprove the reverse containment.

Letting $A = \{1,2\}$, $B = \{2,3\}$, and $U = \{1,2,3,4\}$,
we see $(A^c \setminus B^c) = \{3\}$,
while $(A \setminus B)^c = \{2,3,4\}$.

(6) Consider the set $S = \{1, 2, 3\}$. Locate a partition P of $S \times S$ into exactly 3 parts A_1, A_2, A_3 so that:

- A_1 is a reflexive relation on S;
- A_2 is an irreflexive relation on S; and
- A_3 is a symmetric relation on S.

Briefly justify (e.g., 1 sentence) why your chosen partition is a partition (you do **not** need to justify why each part has the property it does).

$P = \{A_1, A_2, A_3\}$, where

$A_1 = \{(1,1), (2,2), (3,3)\}$,

$A_2 = \{(1,3), (3,1), (2,3), (3,2)\}$

$A_3 = \{(1,2), (2,1)\}$.

Since every element of $S \times S$ lies in exactly one of the A_i (i.e., $A_i \cap A_j = \emptyset$ whenever $i \neq j$, and $A_1 \cup A_2 \cup A_3 = S \times S$), and each of the A_i is <u>non empty</u>, P is a partition.

(7) Let R be a transitive relation on a set S. Prove that R^{-1} is transitive.

Fix $(a,b) \in R^{-1}$ and $(b,c) \in R^{-1}$. We wish to show $(a,c) \in R^{-1}$.
By the definition of R^{-1}, $(b,a) \in R$ and $(c,b) \in R$.
Since R is transitive, $(c,a) \in R$, so $(a,c) \in R^{-1}$ as desired 10

(8) Let $A = \{ab : a, b \in \mathbb{Z}, a > 2b\}$. Prove or disprove: $A = \mathbb{Z}$.

This statement is true.

Proof: first, fix $x \in A$. Then $x = ab$ for some $a, b \in \mathbb{Z}$, so $x \in \mathbb{Z}$.

Next, fix $x \in \mathbb{Z}$. We wish to find $a, b \in \mathbb{Z}$ with $a > 2b$ such that $x = ab$. We proceed by cases.

- If $x \leq 0$, then choosing $b = -1$ and $a = -x$, we see $x = ab$ and $2b = -2 < 0 \leq -x = a$.
- If $x \geq 3$, then choosing $b = 1$ and $a = x$, we see $x = ab$ and $2b = 2 < 3 \leq x = a$.
- If $x = 1$, then choose $a = b = -1$, so $x = ab$ and $2b = -2 < -1 = a$.
- If $x = 2$, then choosing $b = -2$ and $a = -1$, we have $x = ab$ and $2b < a$.

In all cases, $x \in A$. 9