Math 245 - Spring 2026
Instructor: Christopher O'Neill
Wednesday, April 15th, 2026
Midterm Exam 3

Name: ______________________________ **Red ID:** ____________________

Please write legibly, with plenty of white space. Please **print** your name and Red ID in the designated spaces above. Be sure to read problem directions carefully. Please fit your answers into the designated areas; material outside the designated areas (such as on this cover page) will not be graded. Each problem is worth 10-20 points, for a maximum score of 100. The use of notes, books, calculators, or other materials on this exam is strictly prohibited, except you may bring one 3"x5" card (both sides) with your handwritten notes. This exam will begin at 10:00 and will end at 10:50; pace yourself accordingly.

Logistical instructions:

(1) Stow all bags/backpacks/purses at the front of the room. All contraband, except phones, must be stowed in your bag. All smartwatches and phones must be silent, non-vibrating, and either in your pocket or stowed in your bag.
(2) Please remain quiet to ensure a good test environment for others.
(3) Please keep your exam on your desk; do not lift it up for a better look.
(4) If you have a question or need to use the restroom, please come to the front. Bring your exam. I cannot come to you unless you are sitting by an aisle.
(5) If you are done and want to submit your exam and leave, please wait until one of the three designated exit times below. Please do **NOT** leave at any other time. If you are sure you are done, just sit and wait until the next exit time, with this cover sheet visible.

Good luck!

Designated exit times: 10:20 10:40 10:50

(1) Fill in each of the following blanks. No justification is required for this problem.

- Let $A = \{1,2,3\}$, $B = \{1,3,4\}$, and $C = \{1,4\}$. Write the following sets in list notation.

  $(A \cup B)\Delta(A \cap B) = \{$ ____ $\}$

  $(A \cup B) \times (A \cap C) = \{$ ____ $\}$

  $(A \times A) \setminus (B \times B) = \{$ ____ $\}$

- Let $A = \{1,2\}$, $B = 2^A$, and $C = 2^B$. Then $|C| =$ _______, and an example of an element $D \in C$ with $|D| = 2$ is $D =$ ______________________________.

- If $S = \{1,2,3\}$ and $R = \{(1,2),(2,1),(3,3)\}$, then R does not have the ____________ property since $(1,2),(2,1) \in R$, and R does not have the ____________ property since $(2,3),(3,2) \notin R$.

- An example of a relation R on the set $S = \{1,2,3\}$ that is irreflexive and transitive with $|R| = 3$, written in list notation, is $R = \{$ ______________________________________ $\}$.

(2) Write "Yes" next to each of the following sets that has the same cardinality as $\mathbb{N}$, and "No" next to each set that does not. No justification is required for this problem.

_______ $\mathbb{Z} \times \mathbb{Q}$ _______ $\mathbb{Q} \setminus \mathbb{Z}$ _______ $2^{\{1,2,3,4\}}$ _______ $2^{\mathbb{Z}} \setminus 2^{\mathbb{N}}$ _______ $\mathbb{N} \times \mathbb{R}$

(3) Let $S = \{1,2,3,4\}$, and consider the relation

$$R = \{(1,2),(2,3),(3,1),(4,4)\}$$

on S. Fill in the following sets using list notation. You are not required to justify your answers.

Reflexive closure of R: $R \cup \{$ ____ $\}$

Symmetric closure of R: $R \cup \{$ ____ $\}$

Transitive closure of R: $R \cup \{$ ____ $\}$

(4) Let $R = \{4a : a \in \mathbb{Z}\}$, $S = \{5b : b \in \mathbb{Z}\}$, and $T = \{20c : c \in \mathbb{Z}\}$. Fill in the blanks in the following proof that $R \cap S = T$.

Proof: We first prove ______________. Fix $x \in T$. This means $x = 20c$ for some $c \in \mathbb{Z}$.

Choosing $a =$ _______ and $b =$ _______, we see

$$x = \underline{\hspace{4cm}} = 4a \qquad \text{and} \qquad x = \underline{\hspace{4cm}} = 5b$$

so $x \in R$ and $x \in S$. As such, $x \in R \cap S$.

Next, we prove ______________. Fix $x \in R \cap S$, so $x = 4a$ and $x = 5b$ for some $a, b \in \mathbb{Z}$.

In particular, $4a = 5b$. Since ______________________ and 5 is ____________,

we must have $a =$ _______ for some $c \in \mathbb{Z}$. Upon verifying

$$x = \underline{\hspace{4cm}} = 20c,$$

we conclude $x \in T$.

(5) Fix two subsets A and B of a universe set U.

(a) Prove $(A^c \setminus B^c) \subseteq (A \setminus B)^c$.

(b) Disprove the reverse containment.

(6) Consider the set $S = \{1, 2, 3\}$. Locate a partition P of $S \times S$ into exactly 3 parts A_1, A_2, A_3 so that:

- A_1 is a reflexive relation on S;
- A_2 is an irreflexive relation on S; and
- A_2 is a symmetric relation on S.

Briefly justify (e.g., 1 sentence) why your chosen partition is a partition (you do **not** need to justify why each part has the property it does).

(7) Let R be a transitive relation on a set S. Prove that R^{-1} is transitive.

(8) Let $A = \{ab : a, b \in \mathbb{Z}, a > 2b\}$. Prove or disprove: $A = \mathbb{Z}$.