

**Spring 2026, Math 590: Week 3 Preliminary Problems**  
**Due: Wednesday, February 11th, 2026**  
**Generating Functions - Deep Cuts**

**Preliminary problems.** These problems should be completed before discussion on Wednesday.

(P1) Use the fact that

$$\sum_{n \geq 0} n^2 z^n = \frac{z + z^2}{(1 - z)^3}$$

and formal differentiation to verify that

$$\sum_{n \geq 0} n^3 z^n = \frac{z + 4z^2 + z^3}{(1 - z)^4}.$$