

Spring 2026, Math 590: Week 4 Preliminary Problems
Due: Wednesday, February 18th, 2026
Quasipolynomials and Rational Generating Functions

Preliminary problems. These problems should be completed before discussion.

(P1) Suppose

$$\sum_{n \geq 0} a_n z^n = \frac{Q(z)}{(1-z)^{d+1}}.$$

Fill in the table below.

Conditions on a_n	Conditions on $Q(z)$
a_n is _____	$Q(z)$ is a polynomial with $\deg Q(z) < d + 1$
a_n is a polynomial of degree exactly d leading coefficient: _____	$Q(z)$ additionally satisfies _____

(P2) Suppose

$$\sum_{n \geq 0} a_n z^n = \frac{R(z)}{(1-z^p)^{d+1}}.$$

Fill in the table below.

Conditions on a_n	Conditions on $R(z)$
a_n is a quasipolynomial of degree at most d and period dividing p	$R(z)$ is _____
a_n is a polynomial of degree at most d	$R(z)$ is additionally divisible by _____

(P3) We saw in class that if

$$\sum_{n \geq 0} a_n z^n = \frac{1}{(1-z^3)(1-z^5)}$$

then a_n is a quasipolynomial of degree 1 and period 15. Rewrite the right hand side so it matches the theorem at the end of class.