

**Winter 2017, Math 148**  
**Final Exam Review**

The problems below are intended to help you review for the final exam, and may *not* be turned in for credit.

(ER1) Determine whether or not each of the following block designs exists. For those that do, specify a method to construct one and find the number of blocks it would have. If the number of blocks is at most 10, find the design explicitly.

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|---------------------------------|-----------------------------|
| (a) A 2-design (13, 4, 1).      | (d) A 1-design (6, 4, 5).   |
| (b) A 2-design (49, 17, 1).     | (e) A 1-design (6, 4, 6).   |
| (c) A 2-design (10201, 101, 1). | (f) A 7-design (17, 17, 1). |

(ER2) Consider the set  $A = \{0, 1, 2, 4\} \subset \mathbb{Z}_7$ .

- (a) Verify that  $A$  is a difference set.
- (b) Construct a 2-design using  $A$ , and specify its parameters.
- (c) Find the complementary 2-design to the design in part (b). Specify its parameters.
- (d) View your designs in parts (b) and (c) as 1-designs and find their parameters.

(ER3) Determine which of the following codes are linear, and find the value of  $\delta$  for each.

- (a)  $\{000000, 011010, 101101, 110011\} \subset V^6$ .
- (b)  $\{000000, 011010, 100101, 111111\} \subset V^6$ .
- (c)  $\{00000, 10000, 01000, 00100, 00010, 00001\} \subset V^5$ .

(ER4) What is the maximum dimension of a linear code  $C \subset V^8$  that can correct 2 errors? Find a code with this property (you may give a basis, specify a check matrix, or simply list all of the codewords).

(ER5) Consider the code  $C$  defined by the following check matrix.

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

- (a) Find a basis for  $C$ .
- (b) Determine whether  $C$  is guaranteed to correct at least one error.
- (c) Suppose you receive a code 11111111. Is this a valid codeword in  $C$ ? If not, which codeword(s) can it correct to?

(ER6) Is the set

$$R = \{x^n + a_{n-1}x^{n-1} \cdots + a_1x + a_0 : a_0, \dots, a_{n-1} \in \mathbb{Q}\} \subset \mathbb{Q}[x]$$

a ring? If so, is it a field?

(ER7) Determine whether the polynomial  $x^3 + 5x^2 + 3x + 4 \in \mathbb{Q}[x]$  is irreducible.

(ER8) True or false: there is more than one field with infinitely many elements.

(Bonus) Find all codewords for the cyclic code  $C \subset V^4$  corresponding to the ideal  $\langle x + 1 \rangle$ .