Winter 2018, Math 148
Final Exam Review

The problems below are intended to help you review for the final exam, and may *not* be turned in for credit.

(ER1) Determine whether or not each of the following block designs exists. For those that do, specify a method to construct one and find the number of blocks it would have. If the number of blocks is at most 10, find the design explicitly.

(a) A 2-design $(13, 4, 1)$.

(b) A 2-design $(49, 17, 1)$.

(c) A 2-design $(10201, 101, 1)$.

(d) A 1-design $(6, 4, 5)$.

(e) A 1-design $(6, 4, 6)$.

(f) A 7-design $(17, 17, 1)$.

(ER2) Consider the set $A = \{0, 1, 2, 4\} \subset \mathbb{Z}_7$.

(a) Verify that A is a difference set.

(b) Construct a 2-design using A, and specify its parameters.

(c) Find the complementary 2-design to the design in part (b). Specify its parameters.

(d) View your designs in parts (b) and (c) as 1-designs and find their parameters.

(ER3) Determine which of the following codes are linear, and find the value of δ for each.

(a) $\{000000, 011010, 101101, 110011\} \subset V^6$.

(b) $\{000000, 011010, 100101, 111111\} \subset V^6$.

(c) $\{00000, 10000, 01000, 00100, 00010, 00001\} \subset V^5$.

(ER4) What is the maximum dimension of a linear code $C \subset V^8$ that can correct 2 errors? Find a code with this property (you may give a basis, specify a check matrix, or simply list all of the codewords).

(ER5) Consider the code C defined by the following check matrix.

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

(a) Find a basis for C.

(b) Determine whether C is guaranteed to correct at least one error.

(c) Suppose you receive a code 11111111. Is this a valid codeword in C? If not, which codeword(s) can it correct to?

(ER6) Is the set

$$R = \{x^n + a_{n-1}x^{n-1} \cdots + a_1 x + a_0 : a_0, \ldots, a_{n-1} \in \mathbb{Q}\} \subset \mathbb{Q}[x]$$

a ring? If so, is it a field?

(ER7) Determine whether the polynomial $x^3 + 5x^2 + 3x + 4 \in \mathbb{Q}[x]$ is irreducible.

(ER8) True or false: there is more than one field with infinitely many elements.

(Bonus) Find all codewords for the cyclic code $C \subset V^4$ corresponding to the ideal $\langle x + 1 \rangle$.