

Math 16B, Section 3 - Winter 2018
Instructor: Christopher O'Neill
Practice Final Exam, Version 1

Last Name: _____ **First Name:** _____

Directions:

- The use of a calculator, cell phone, laptop or computer is prohibited.
- TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
- Answer all of the questions, and present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer*. You will be graded not merely on the final answer, but on the quality and correctness of the work leading up to it.

The UC Davis Code of Academic Conduct

I will conduct myself with honesty, fairness, and integrity.

Signature: _____

(1) Find the derivative of each of the following functions.

(a) $f(x) = 3x^2 + 2x + x \sin(x)$

(b) $f(x) = 3e^{2x} + 4x^2e^{5x}$

(c) $f(x) = \ln(x^2 + 2x + 1)$

(2) Solve for x in the following equation.

$$\ln(x) + \ln(4) = \ln(x + 3)$$

(3) Suppose you invest \$5000 in a savings account with an interest rate of 5%, compounded monthly.

(a) How much money will be in the account after 5 years?

(b) Suppose that instead of compounding monthly, the interest is compounded continuously. How long will it take for the account balance to reach \$6000?

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(4) Suppose a colony of bacteria initially has 1000 bacteria and grows to 1500 bacteria after 5 hours.

(a) Use an exponential growth model to find $P(t)$, the population after t hours.

(b) Based on your model in part (a), what will the population be after 24 hours?

(c) How long does it take the colony to double its population?

- (5) Approximate the following integral using trapezoid rule with $n = 4$.

$$\int_0^{\pi} \sin(x) \, dx$$

- (6) A skydiver jumps from a plane at 12000ft. Assuming they have not yet deployed their parachute, how long does it take the skydiver to reach ground level? Assume the velocity is given by

$$v(t) = -32t.$$

- (7) Find the total area of the bounded region(s) between the following curves.

$$f(x) = x^2 \quad \text{and} \quad g(x) = 4x$$

(8) Evaluate each of the following integrals.

(a) $\int 6x^2 e^{2x} dx$

(b) $\int \frac{\cos(\ln(x))}{x} dx$

(c) $\int x^2 \sin(x) dx$

$$(d) \int \frac{x^3 + 2}{x^2 - 1} dx$$

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$$(e) \int_0^{\pi/2} \frac{\cos(x)}{1 + \sin(x)} dx$$

$$(f) \int_0^{\infty} 4xe^{-x^2} dx$$

(9) Suppose you have a coin with “1” on one side and “3” on the other. Consider flipping this coin 3 times, and let x denote the random variable that records the sum of the values that come up.

(a) List the elements of the sample space.

(b) What is the probability that $x = 3$?

(c) Compute the mean of x .

(d) Compute the variance of x .

- (10) Consider the continuous random variable x that takes values in the range $0 \leq x \leq 2$ with

$$f(x) = kx^2$$

as a probability density function.

- (a) Find the value for k so that $f(x)$ is a probability density function.

- (b) Find $P(1 \leq x \leq 2)$.

- (c) Find the median of x .

- (d) Find the variance of x .

Trigonometric Identities

$$\sin(A + B) = \sin(A) \cos(B) + \cos(A) \sin(B)$$

$$\sin(A - B) = \sin(A) \cos(B) - \cos(A) \sin(B)$$

$$\cos(A + B) = \cos(A) \cos(B) - \sin(A) \sin(B)$$

$$\cos(A - B) = \cos(A) \cos(B) + \sin(A) \sin(B)$$

$$\sin(2A) = 2 \sin(A) \cos(A)$$

$$\cos(2A) = \cos^2(A) - \sin^2(A)$$

$$\sin^2(A) + \cos^2(A) = 1$$

$$\tan^2(A) + 1 = \sec^2(x)$$

$$1 + \cot^2(A) = \csc^2(x)$$

$$\int \sec(x) \, dx = \ln |\sec(x) + \tan(x)| + C$$

$$\int \csc(x) \, dx = -\ln |\csc(x) + \cot(x)| + C$$

Error Estimates

$$|E_T| \leq \frac{M(b-a)^3}{12n^2} \quad f''(x) \leq M \text{ for all } x \in [a, b]$$