Math 16B, Section 3 - Winter 2018
Instructor: Christopher O'Neill
Practice Final Exam, Version 2

Last Name: _____ **First Name:** _____

Directions:

- The use of a calculator, cell phone, laptop or computer is prohibited.
- You may bring to the exam a double-sided index card whose dimensions do not exceed 4in x 6in. Violating this rule will result in a 0 on the exam.
- TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
- Answer all of the questions, and present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but on the quality and correctness of the work leading up to it.

The UC Davis Code of Academic Conduct

I will conduct myself with honesty, fairness, and integrity.

Signature: _____

(1) Find the *derivative* of each of the following functions.

(a) $f(x) = \sin(x) + \cos(x) + x^6$

(b) $f(x) = 3e^{3x^2}$

(c) $f(x) = \ln(1 + e^x)$

(2) Write the following expression using only a single logarithm.

$$\ln(x+1) + \ln(x-1) - 4\ln(x)$$

(3) Suppose you invest \$7000 in a savings account with an interest rate of 5%, compounded daily.

(a) Find the account balance after 7 years.

(b) How long will it take for the account balance to reach \$10,000?

(4) Suppose you find 100g of a radioactive substance with a half life of 1000 years.

(a) Find an exponential decay model for $R(t)$, the number of grams of substance left after t years.

(b) How much of the substance is left after 2000 years?

(5) Approximate the following integral using midpoint rule with $n = 3$.

$$\int_1^7 (x^2 + 2x + 1)\, dx$$

(6) Solve the following initial value problem.

$$f''(x) = 6x + 2, \qquad f'(0) = 1, \qquad f(1) = 2$$

(7) Find the total area of the bounded region(s) between the following curves.

$$f(x) = \cos(x), \qquad g(x) = 1, \qquad x = 0, \qquad \text{and} \qquad x = 2\pi$$

(8) Evaluate each of the following integrals.

(a) $\int 4xe^{-2x}\ dx$

(b) $\int \frac{x\cos(x^2)}{1+\sin(x^2)}\ dx$

(c) $\int x^3 \ln(x)\ dx$

(d) $\displaystyle\int \frac{1}{(x-2)(x+3)^2}\,dx$

(e) $\displaystyle\int_0^{\pi} \cos(x)\sin(\sin(x))\ dx$

(f) $\displaystyle\int_0^{\infty} \frac{e^{2x}}{e^{2x}+1}\ dx$

(9) Suppose you roll a standard 6-sided die (once). Let x denote the random variable that records the square of the value that comes up (for instance, if a 3 is rolled, the value of x is 9).

(a) List the elements of the sample space, and indicate the corresponding value of x for each.

(b) Compute the mean of x.

(c) Compute the variance of x.

(10) Consider the continuous random variable x that takes values in the range $0 \leq x \leq 4$ with

$$f(x) = \frac{3}{32}x(4-x)$$

as a probability density function.

(a) Verify that $f(x)$ is indeed a probability density function.

(b) Find $P(1 \leq x \leq 3)$.

(c) Find the mean x.

(d) Find the median of x.

Trigonometric Identities

$$\begin{aligned}
\sin(A+B) &= \sin(A)\cos(B)+\cos(A)\sin(B)\\
\sin(A-B) &= \sin(A)\cos(B)-\cos(A)\sin(B)\\
\cos(A+B) &= \cos(A)\cos(B)-\sin(A)\sin(B)\\
\cos(A-B) &= \cos(A)\cos(B)+\sin(A)\sin(B)
\end{aligned}$$

$$\begin{aligned}
\sin(2A) &= 2\sin(A)\cos(A)\\
\cos(2A) &= \cos^2(A)-\sin^2(A)
\end{aligned}$$

$$\begin{aligned}
\sin^2(A)+\cos^2(A) &= 1\\
\tan^2(A)+1 &= \sec^2(x)\\
1+\cot^2(A) &= \csc^2(x)
\end{aligned}$$

$$\begin{aligned}
\int \sec(x)\,dx &= \ln|\sec(x)+\tan(x)|+C\\
\int \csc(x)\,dx &= -\ln|\csc(x)+\cot(x)|+C
\end{aligned}$$

Error Estimates

$$|E_T| \leq \frac{M(b-a)^3}{12n^2} \qquad f''(x)\leq M \text{ for all } x\in[a,b]$$