

Math 16B: Short Calculus II
 Winter 2018, Section 3
 Homework Sheet 2
 Due: Monday, January 22, 2018

Submit your solutions to the following problems in lecture on the due date above. Present your work in a clean and organized fashion, either on a printed copy of this document (preferred) or a separate sheet of paper. As stated in the syllabus, late submissions will not be accepted.

1. Find the derivatives of the following functions.

(a) $f(x) = \frac{\ln(x)}{e^x}$ *quotient rule*

$$f'(x) = \frac{\frac{1}{x}e^x - \ln(x)e^x}{(e^x)^2}$$

(b) $f(x) = \ln(x^2(x+1)^3)$ *chain rule*

$$f'(x) = \frac{1}{x^2(x+1)^3} \cdot \frac{d}{dx} [x^2(x+1)^3] \text{ *product rule*}$$

$$= \frac{1}{x^2(x+1)^3} (2x(x+1)^3 + x^2(3(x+1)^2(1)))$$

2. Using properties of logarithms, write the following using only a single logarithm.

$$3(\ln(x+2) - 4\ln(2x^3) + \ln(x^2+1))$$

~~$3(\ln(x+2) - 4\ln(2x^3) + \ln(x^2+1))$~~

$$= 3(\ln(x+2) - \ln((2x^3)^4) + \ln(x^2+1))$$

$$= 3\left(\ln\left(\frac{x+2}{(2x^3)^4}\right) + \ln(x^2+1)\right)$$

$$= \ln\left(\frac{(x+2)(x^2+1)}{(2x^3)^4}\right)$$

$$= \ln\left[\left(\frac{(x+2)(x^2+1)}{(2x^3)^4}\right)^3\right]$$

3. Find the half-life of a radioactive substance for which 99% remains after 1 year.

$$R(t) = Ce^{-kt}$$

$C = \text{initial amount}$

$$R(1) = 0.99C$$

$$0.99C = Ce^{-k(1)}$$

$$0.99 = e^{-k}$$

$$\ln(0.99) = -k$$

$$|k = -\ln(0.99)$$

$$R(t) = Ce^{t \cdot \ln(0.99)}$$

$$\frac{1}{2}C = Ce^{t \ln(0.99)}$$

$$\frac{1}{2} = e^{t \ln(0.99)}$$

$$\ln\left(\frac{1}{2}\right) = t \cdot 0.99$$

$$t = \frac{\ln(1/2)}{\ln(0.99)} \text{ yrs}$$