Math 16B: Short Calculus II
Winter 2018, Section 3
Homework Sheet 4
Due: Monday, February 12, 2018

Submit your solutions to the following problems in lecture on the due date above. Present your work in a clean and organized fashion, either on a printed copy of this document (preferred) or a separate sheet of paper. As stated in the syllabus, late submissions will **not** be accepted.

1. Evaluate the following integrals.

(a) $\int_3^6 (x^2 + 2x + 1)\, dx = \frac{1}{3}x^3 + x^2 + x \Big|_{x=3}^{x=6}$

$= \left(\frac{1}{3}(6)^3 + (6)^2 + 6\right) - \left(\frac{1}{3}(3)^3 + (3)^2 + 3\right)$

$= (72 + 36 + 6) - (9 + 9 + 3) = 114 - 21 = \boxed{93}$

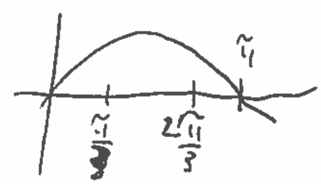
(b) $\int_0^\pi 21\cos(7x)\, dx = 3\sin(7x)\Big|_{x=0}^{x=\pi} = 3\sin(7\pi) - 3\sin(0) = \boxed{0}$

check: $\frac{d}{dx}\left[3\sin(7x)\right] = 3\cos(7x)\cdot 7$

$= 21\cos(7x)$

2. Consider the following integral.

$$\int_0^\pi \sin(x)\, dx$$



(a) Approximate the above integral using a midpoint sum with $n = 3$ subdivisions.

$\left[\sin\left(\frac{\pi}{6}\right)\right]\cdot\left(\frac{\pi}{3}\right) + \left[\sin\left(\frac{\pi}{2}\right)\right]\left(\frac{\pi}{3}\right) + \left[\sin\left(\frac{5\pi}{6}\right)\right]\left(\frac{\pi}{3}\right)$

$= \left(\frac{1}{2}\right)\left(\frac{\pi}{3}\right) + (1)\left(\frac{\pi}{3}\right) + \left(\frac{1}{2}\right)\left(\frac{\pi}{3}\right) = \boxed{\frac{2\pi}{3}}$

(b) Compare your estimate to the exact area under the curve.

$\int_0^\pi \sin(x)\, dx = -\cos(x)\Big|_{x=0}^{x=\pi} = (-\cos(\pi)) - (-\cos(0)) = 1 - (-1) = \boxed{2}$

Error $= \left|2 - \frac{2\pi}{3}\right|$.