Math 16B: Short Calculus II
Winter 2018, Section 3
Homework Sheet 6
Due: Wednesday, February 28, 2018

Submit your solutions to the following problems in lecture on the due date above. Present your work in a clean and organized fashion, either on a printed copy of this document (preferred) or a separate sheet of paper. As stated in the syllabus, late submissions will **not** be accepted.

1. Evaluate the following integrals.

(a) $\int x^2 e^{5x}\,dx$

$= \frac{1}{5}x^2e^{5x} - \int \frac{2}{5}xe^{5x}\,dx = \frac{1}{5}x^2e^{5x} - \left[\frac{2}{25}xe^{5x} - \int \frac{2}{25}e^{5x}\,dx\right]$

$= \frac{1}{5}x^2e^{5x} - \frac{2}{25}xe^{5x} + \frac{2}{125}e^{5x}$

$u = x^2 \quad v = \frac{1}{5}e^{5x}$

$\frac{du}{dx} = 2x \quad \frac{dv}{dx} = e^{5x}$

$du = 2x\,dx \quad dv = e^{5x}\,dx$

$u = \frac{2}{5}x \quad v = \frac{1}{5}e^{5x}$

$\frac{du}{dx} = \frac{2}{5} \quad \frac{dv}{dx} = e^{5x}$

$du = \frac{2}{5}\,dx \quad dv = e^{5x}\,dx$

(b) $\int x^3\cos(x)\,dx$

$= x^3\sin(x) - \int 3x^2\sin(x)\,dx = x^3\sin(x) - \left[-3x^2\cos(x) + \int 6x\cos(x)\,dx\right]$

$= x^3\sin(x) - \left[-3x^2\cos(x) + 6x\sin(x) + 6\cos(x) + C\right]$

$u = x^3 \quad v = \sin(x)$

$\frac{du}{dx} = 3x^2 \quad \frac{dv}{dx} = \cos(x)$

$du = 3x^2\,dx \quad dv = \cos(x)\,dx$

$u = 3x^2 \quad v = -\cos(x)$

$\frac{du}{dx} = 6x \quad \frac{dv}{dx} = \sin(x)$

$du = 6x\,dx \quad dv = \sin(x)\,dx$

$u = 6x \quad v = \sin(x)$

$\frac{du}{dx} = 6 \quad \frac{dv}{dx} = \cos(x)$

$du = 6\,dx \quad dv = \cos(x)\,dx$

(c) $\int x^2\ln(x^3)\,dx$

$= \int 3x^2\ln(x)\,dx = x^3\ln(x) - \int x^3\cdot\frac{1}{x}\,dx = x^3\ln(x) - \int x^2\,dx$

$= x^3\ln(x) - \frac{1}{3}x^3 + C$

$u = \ln(x) \quad v = x^3$

$du = \frac{1}{x}\,dx \quad dv = 3x^2\,dx$

(d) $\int \frac{x^2-3}{(x^2-1)(x+1)}\,dx$

$= \int \frac{x^2-3}{(x-1)(x+1)^2}\,dx = \int \left(\frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}\right)dx$

$x^2 - 3 = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$

$x = 1: \quad -2 = 4A + 0 + 0 \quad A = -2$

$x = -1: \quad -2 = 0 + 0 - 2C \quad C = 1$

$x^2 - 3 = -2(x+1)^2 + B(x-1)(x+1) + (x-1)$

$x = 0: \quad -3 = -2 - B - 1 \quad B = 0$

$= \int \left(\frac{-2}{x-1} + \frac{1}{(x+1)^2}\right)dx$

$= -2\ln|x-1| + -(x+1)^{-1} + C$