Math 16B: Short Calculus II
Winter 2018, Section 3
Homework Sheet 8
Due: Friday, March 16, 2018

Submit your solutions to the following problems in lecture on the due date above. Present your work in a clean and organized fashion, either on a printed copy of this document (preferred) or a separate sheet of paper. As stated in the syllabus, late submissions will **not** be accepted.

1. Suppose you have a 4-sided die with side labels 1, 2, 2, and 4. Consider the (discrete) random variable x that counts the number of even values that occur when rolling it twice.

   (a) Identify all possible outcomes in the sample space, and find the probability of each.

   $p(11) = \frac{1}{16}$ $\quad p(21) = \frac{2}{16}$ $\quad p(41) = 1/16$

   $p(12) = \frac{2}{16}$ $\quad p(22) = 4/16$ $\quad p(42) = 2/16$

   $p(14) = \frac{1}{16}$ $\quad p(24) = 2/16$ $\quad p(44) = 1/16$

   (b) Find the expected value (i.e. mean), variance, and standard deviation of x.

   Mean: $\mu = (0)\underbrace{(1/16)}_{P(X=0)} + (1)\underbrace{(6/16)}_{P(X=1)} + (2)\underbrace{(9/16)}_{P(X=2)} = 24/16 = \boxed{1.5}$

   variance: $V = (0-1.5)^2\underbrace{(1/16)}_{p(x=0)} + (1-1.5)^2\underbrace{(6/16)}_{p(x=1)} + (2-1.5)^2\underbrace{(9/16)}_{p(x=2)} = \boxed{0.375}$

   Standard deviation: $\sigma = \sqrt{V} \approx \boxed{0.612}$

2. Let x be a continuous random variable with probability density function

$$f(x) = k\sin(x)$$

   for $0 \le x \le \pi$.

   (a) Find a value of k so that f is a probability density function.

   $\int_0^{\pi} k\sin(x)\,dx = -k\cos(x)\Big|_{x=0}^{x=\pi} = (-k(-1)) - (-k(1)) = 2k = 1$

   $\boxed{k = \frac{1}{2} \quad f(x) = \frac{1}{2}\sin(x)}$

   (b) Find the expected value (i.e. mean) and median of f.

   mean: $\mu = \int_0^{\pi} x\left(\frac{1}{2}\sin(x)\right)dx = -\frac{1}{2}x\cos(x) - \int_0^{\pi} -\frac{1}{2}\cos(x)\,dx = -\frac{1}{2}x\cos(x) + \frac{1}{2}\sin(x)\Big|_0^{\pi}$

   $u = x \quad v = -\frac{1}{2}\cos(x)$

   $du = dx \quad dv = \frac{1}{2}\sin(x)\,dx$

   $= \left[-\frac{1}{2}\pi(-1) + \frac{1}{2}(0)\right] - \left[-\frac{1}{2}(0)(1) + \frac{1}{2}(0)\right]$

   $= \boxed{\frac{\pi}{2}}$

   median: Find M so $\int_0^{M} \frac{1}{2}\sin(x)\,dx = \frac{1}{2}$

   $\int_0^{M} \frac{1}{2}\sin(x)\,dx = -\frac{1}{2}\cos(x)\Big|_0^{M} = \left[-\frac{1}{2}\cos(M)\right] - \left[-\frac{1}{2}\cos(0)\right] = \frac{1}{2}$

   $-\frac{1}{2}\cos(M) + \frac{1}{2} = \frac{1}{2}$

   $\cos(M) = 0 \qquad M = \boxed{\frac{\pi}{2}}$